MEMBRANE ADAPTIVE OPTICS (Preprint)

Dan Marker, James Wilkes, Eric Riggiero, Dab Inman

1 August 2005

Technical Paper/Conference Proceedings

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION IS UNLIMITED.



AIR FORCE RESEARCH LABORATORY
Directed Energy Directorate
3550 Aberdeen Ave SE
AIR FORCE MATERIEL COMMAND
KIRTLAND AIR FORCE BASE, NM 87117-5776

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)		
01-08-2005	Conference Proceedings	01 Oct 97 - 01 Aug 05		
4. TITLE AND SUBTITLE		5a. CONTRACT NUMBER		
Membrane Adaptive Optics (Preprint)		DF297347		
		5b. GRANT NUMBER		
		5c. PROGRAM ELEMENT NUMBER		
		65502C		
6. AUTHOR(S)	5d. PROJECT NUMBER			
Dan Marker, James Wilkes,	Eric Riggiero, Dab Inman	5031		
		5e. TASK NUMBER		
		SP		
		5f. WORK UNIT NUMBER		
		25		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)		8. PERFORMING ORGANIZATION REPORT NUMBER		
AFRL/DESE				
3550 Aberdeen Avenue SE				
Kirtland AFB, NM 87117-5776				
, , , , , , , , , , , , , , , , , , , ,				
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR'S ACRONYM(S)		
		AFRL/DES		
AIR FORCE RESEARCH LAB	ORATORY			
3550 Aberdeen Ave. SE		11. SPONSOR/MONITOR'S REPORT		
Kirtland AFB, NM 87117-5776		NUMBER(S)		
Tandana / a B, Mill Of 117-3770	,	AFRL-DE-PS-TP-2006-1008		
40 DIOTRIBUTION / AVAIL ABILITY OF A				

12. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for Public Release; distribution unlimited. (Public Release Approval No. 05-259.)

13. SUPPLEMENTARY NOTES

Submitted for publication in SPIE 50th Annual Meeting, Photonics. GOVERNMENT PURPOSE RIGHTS.

14. ABSTRACT

An innovative adaptive optic is discussed that provides a range of capabilities unavailable with either existing, or newly reported, research devices. It is believed that this device will be inexpensive and uncomplicated to construct and operate, with a large correction range that should dramatically relax the static and dynamic structural tolerances of a telescope. As the areal density of a telescope primary is reduced, the optimal optical figure and the structural stiffness are inherently compromised and this phenomenon will require a responsive, range-enhanced wavefront corrector. In addition to correcting for the aberrations in such innovative primary mirrors, sufficient throw remains to provide non-mechanical steering to dramatically improve the field of regard. Time dependent changes such as thermal disturbances can also be accommodated. The proposed adaptive optic will overcome some of the issues facing conventional deformable mirrors, as well as current and proposed MEMS-based deformable mirrors and liquid crystal based adaptive optics. Such a device is scalable to meter diameter apertures, eliminates high actuation voltages with minimal power consumption, provides long throw optical path correction, provides polychromatic dispersion free operation, dramatically reduces the effects of adjacent actuator influence, and provides a nearly 100% useful aperture. This article will reveal top-level details of the proposed construction and include portions of a static, dynamic, and residual aberration analysis. This device will enable certain designs previously conceived by visionaries in the optical community.

15. SUBJECT TERMS

Optical, adaptive optic, membrane, large, polymer

16. SECURITY CLASSIFICATION OF:		17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON Dan Marker	
a. REPORT Unclassified	b. ABSTRACT Unclassified	c. THIS PAGE Unclassified	SAR	10	19b. TELEPHONE NUMBER (include area code) 505-846-2871

Membrane Adaptive Optics

Dan K. Marker^a, James M. Wilkes^a, Eric J. Ruggiero^b, and Daniel J. Inman^b

^aAFRL/Directed Energy Directorate, 3550 Aberdeen SE, Kirtland AFB, NM 87117, USA

^bCenter for Intelligent Material Systems and Structures,

Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA

ABSTRACT

An innovative adaptive optic is discussed that provides a range of capabilities unavailable with either existing, or newly reported, research devices. It is believed that this device will be inexpensive and uncomplicated to construct and operate, with a large correction range that should dramatically relax the static and dynamic structural tolerances of a telescope. As the areal density of a telescope primary is reduced, the optimal optical figure and the structural stiffness are inherently compromised and this phenomenon will require a responsive, range-enhanced wavefront corrector. In addition to correcting for the aberrations in such innovative primary mirrors, sufficient throw remains to provide non-mechanical steering to dramatically improve the field of regard. Time dependent changes such as thermal disturbances can also be accommodated. The proposed adaptive optic will overcome some of the issues facing conventional deformable mirrors, as well as current and proposed MEMS-based deformable mirrors and liquid crystal based adaptive optics. Such a device is scalable to meter diameter apertures, eliminates high actuation voltages with minimal power consumption, provides long throw optical path correction, provides polychromatic dispersion free operation, dramatically reduces the effects of adjacent actuator influence, and provides a nearly 100% useful aperture. This article will reveal top-level details of the proposed construction and include portions of a static, dynamic, and residual aberration analysis. This device will enable certain designs previously conceived by visionaries in the optical community.

Keywords: Optical, adaptive optic, membrane, large, polymer

1. INTRODUCTION

This Adaptive Optic (AO) concept is referred to as a Pressure-augmented, Boundary-controlled, Spatially-actuated (PBS) adaptive mirror. The concept was enabled by products produced from the membrane mirror program which has enabled production of large diameter (10-100 cm) optical quality polyimide based membrane films with dielectric coatings. In this paper we discuss the theoretical aspects of the boundary control of static wavefront aberrations attainable by such a mirror, as well as the stiffening and damping effects of the air gap (see figure 1), and the added-mass effects of the air external to the membrane front surface.

The latter two are fluid/structure interactions that affect the natural vibration frequencies of the membrane; they are competitive in that the stiffening by the air gap would be expected to raise the frequencies (desirable), while the added-mass effect of the external air would be expected to lower the frequencies (undesirable). Our system is unique as the PBS will be operated at very low membrane tension ($T \approx 10-50 \text{ N/m}$, a force per unit circumferential length) in order to (a) reduce the forces required to actuate it, and (b) reduce the load on its boundary ring. Reduction of the actuation and boundary forces lowers the required actuation voltages, leading to lower power requirements and reduction in system weight. In addition to operating in a low tension state, the dimensionless aspect ratio of air gap depth, ℓ , to membrane radius, a, for the PBS is quite small (on the order of $10^{-3} - 10^{-4}$). The two requirements of low tension and low aspect ratio define a regime of fluid/structure interaction that has been for the most part neglected in the existing literature. This interaction must be fully understood as a preliminary to the design of control algorithms for dynamic control of the adaptive mirror in the presence of external noise sources. The results of such a study should also directly benefit researchers in the field of MEMS-based adaptive optics designed to operate under a subset of these conditions.

Correspondence should be addressed to Dan Marker

The possibility of compensating for particular types of static aberration by deforming the boundary ring of a tensioned membrane was suggested in earlier publications by Angel and his co-workers.^{1–4} We provide additional detail here regarding the appropriate shape of the boundary for a given wavefront aberration, and introduce a new set of polynomials intimately related to the Zernike circle polynomials. Unlike the Zernike polynomials, however, each of these polynomials is zero at the membrane edge. Expressing the wavefront aberration in terms of this set yields a significantly lower amplitude residual to be compensated by (nonuniform) spatial actuation (for example, using electrostatic actuators).

The final feature of our concept is the establishment of a uniform (hydrostatic) pressure difference between the front (top) and back (bottom) surfaces of the membrane. This can be achieved by sealing the space between the membrane bottom surface and the backplate, and then either raising or lowering the pressure of the air in this closed volume. As we shall show, such a pressure difference can effectively compensate for any defocus component of the wavefront aberration.

Since the late 1970's, when the earliest attempts at construction of adaptive mirrors utilizing membranes as reflecting elements were made by Grosso and Yellin,⁵ it has been known that the air gap between membrane and backplate affects to some extent the dynamic behavior of the mirror. However, to the present day, this interaction does not appear to be well-understood, and to our knowledge has never been treated systematically with quantifiable results. By systematic and quantifiable, we mean an understanding of the mirror dynamics obtained by application of an appropriate theory. In fact, to our knowledge, the only attempt to explain the effects of the air gap occurred in the original paper by Grosso and Yellin,⁵ where Morse's⁶ theory of the kettledrum (see pp. 193–195 of Morse,⁶ and his Eq. (22.5), p. 221) was used to compute the change in natural frequencies due to the presence of the air gap. In Grosso and Yellin,⁵ experimental results for the damping effects of the air are also discussed, citing only a private communication from M. J. E. Golay as reference. No attempt was made by these authors either to derive equations describing the damping effects due to air viscosity, or to put Morse's kettledrum analysis of the natural frequency changes on a firm theoretical foundation, i.e., to determine under what circumstances his results follow from, say, the Navier-Stokes theory of a viscous fluid. It is thus difficult to determine the applicability of the results in Grosso and Yellin, to systems defined by parameters different from those treated there.

Our intention is to provide a theoretical foundation for the study of these effects, including criteria for determining the applicability of a particular theory to a particular problem. We take as our fundamental premise that the air in the gap and external to the membrane is a viscous gas, described by the Navier-Stokes equations of fluid dynamics. These equations can be put into a non-dimensional form using characteristic values of air pressure, air density, fluid velocity, system geometry (length scales), and a time scale, from which one can, in principle, determine the dominant terms of the equations. The terms retained characterize a particular theory, whose equations must be solved to determine the effects of air damping, stiffening, and added mass on the membrane dynamics. The membrane dynamics are assumed to be governed by the membrane equations of elasticity theory, subject to the loading effects of the air pressure, so the resulting theory will involve the coupling between Navier-Stokes fluid dynamics and membrane mechanics. Our specific goals are to derive the appropriate coupled equations involving the membrane displacement and the air pressure differential induced by this displacement, and solve the equations to obtain the altered natural frequencies and membrane modes. The full details of this analysis are well beyond the page constraint of this article, and our research is still not complete, so what may at times appear to be heuristic arguments are in fact the results of a nearly completed thorough theoretical study. These details will be provided by one of the authors (JMW) to the Air Force Office of Scientific Research (AFOSR) in a Laboratory Annual Report for FY2005 (entitled "Analysis of air gap effects on the dynamics of an adaptive membrane mirror"). We present here only a brief overview of the results of this study.

2. DEVICE DESCRIPTION

No specific hardware has yet been assembled to validate the assertions claimed in this article but a significant amount of related work does support, at various levels, the concepts.⁷ These are enabled primarily by the large area optical quality membranes manufactured by SRS Technologies, Huntsville, AL. The Air Force Research

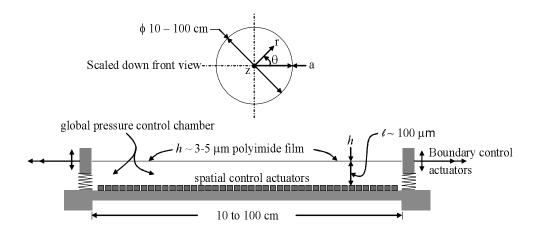


Figure 1: General description of PBS

Laboratory, Directed Energy Directorate (AFRL/DE), supported this development through a series of Small Business Innovative Research (SBIR) contracts. AFRL/DE used a 5-micrometer thick polyimide film to image Saturn, which is the only instance of a membrane being used to image a planet.⁸ A 1-meter diameter film has been manufactured with a thickness variation of 30 nm rms

The primary function of this device is to allow large-area long-throw aberration correction. The degrees-of-freedom of interest are boundary ring displacement, hydrostatic control, and electrostatic control (piston, tip, and tilt will be ignored). Selected components of the wavefront error (defocus, astigmatism, trefoil, and others) can be removed using a combination of extremely low hydrostatic pressure (either positive or negative) and boundary ring displacement. Low voltage electrostatic actuators will be used to eliminate the remaining higher-order aberrations such as coma, third-order spherical aberration, and others. Historically, this approach to wavefront compensation has relied almost solely on the use of electrostatic actuation across a reduced aperture that avoids an annular region near the boundary. Figure 1 shows the general configuration of the PBS.

For convenience, the control authority of the PBS will be separated into four categories:

Control 1: Piston, tip/tilt is a common well-known control authority and is not considered here.

Control 2: Hydrostatic pressure offers the unique ability to identically correct for defocus. Note that it is not necessary to consider an actuator influence function for this correction.

Control 3: Boundary undulation can be implemented using a variety of existing actuation devices such as PZTs and others. This degree-of-freedom allows identical correction for any order of astigmatism. Again, it is not necessary to consider an actuator influence function for this correction.

Control 4: This degree-of-freedom is used by all of the existing MEMS concepts, but for the PBS the vast majority of the aberration is corrected using control authorities 1, 2, & 3, above. This relaxes the higher voltage demands required by MEMS concepts, and significantly reduces the effect of actuator influence by dramatically reducing the magnitude of correction required by the spatial control actuators.

3. STATIC ABERRATION COMPENSATION

We begin with the dynamical equation of motion of an elastic membrane, viz.,

$$\nabla_{r\theta}^2 w + \frac{\Delta p}{T} = \frac{\gamma_0}{T} \frac{\partial^2 w}{\partial t^2}.$$
 (1)

The left-hand side of (1) contains the two-dimensional Laplacian operator in plane polar coordinates acting on the axial displacement $w(r, \theta, t)$:

$$\nabla_{r\theta}^2 w \equiv \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \equiv \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2}. \tag{2}$$

In equation (1) T is the (assumed to be uniform) tension in the membrane, and γ_0 is the areal density (kg/m²) of the membrane, i.e.,

$$\gamma_0 \equiv \rho_m h, \tag{3}$$

where ρ_m is the membrane mass density (kg/m³), h is the membrane thickness, and Δp is the pressure differential across the membrane. A standard derivation of equation (1) can be found in, for example, Graff's textbook.⁹

Suppose, now, that an aberrated wavefront is incident on the membrane reflector. The light is first passed thru a wavefront analyzer to measure its aberrations, and it is assumed that the aberration does not change over the interval of time necessary to deform the membrane to a shape that compensates for some, or all, of the measured aberration. The membrane surface can be changed by either deforming the boundary, or increasing or decreasing the air pressure in the gap. In addition, we suppose that electrostatic actuators providing an additional electrostatic pressure $p_e(r,\theta)$ are used to further deform the membrane in an attempt to remove the small aberrations that remain. Assuming that the aberrations are changing slowly enough for the membrane to achieve a new equilibrium configuration, the dynamic equation (1) simplifies to the time-independent equilibrium equation

$$\nabla_{r\theta}^2 w + \frac{\Delta p + p_e}{T} = 0, \tag{4}$$

where the electrostatic pressure is assumed to enter into the equation in the same way as the hydrostatic pressure difference Δp . It is convenient to introduce a dimensionless radial variable

$$\widehat{r} \equiv \frac{r}{a},\tag{5}$$

in which case the equilibrium equation can be written as

$$\nabla_{\hat{r}\theta}^2 w = -\left(\frac{\Delta p + p_e}{T}\right) a^2,\tag{6}$$

where

$$\nabla_{\widehat{r}\theta}^2 w \equiv \frac{\partial^2 w}{\partial \widehat{r}^2} + \frac{1}{\widehat{r}} \frac{\partial w}{\partial \widehat{r}} + \frac{1}{\widehat{r}^2} \frac{\partial^2 w}{\partial \theta^2} \equiv \frac{1}{\widehat{r}} \frac{\partial}{\partial \widehat{r}} \left(\widehat{r} \frac{\partial w}{\partial \widehat{r}} \right) + \frac{1}{\widehat{r}^2} \frac{\partial^2 w}{\partial \theta^2}, \tag{7}$$

is the Laplacian in dimensionless coordinates. The displacement $w(\hat{r}, \theta)$ required to compensate for a wavefront aberration $W(\hat{r}, \theta)$ incident upon the membrane reflector is simply $w(\hat{r}, \theta) = \frac{1}{2}W(\hat{r}, \theta)$.

Table 1, which follows, compares a short list of Zernike terms to the results of a linear membrane mechanics study. The "CorrectablePortion" column contains shapes that can be identically assumed by amembrane utilizing the first three control modes. For these corrections not a single electrostatic actuator is required, and there is no need to consider actuator influence.

An important feature of the PBS concept is the dramatically reduced electrostatic pressure required to compensate for residual aberrations, since a typical aberration is dominated by components that can be fully compensated with control authorities 1 through 3. Based on a preliminary analysis, voltages on the order of 10 volts should be sufficient to completely remove the total residual aberration. However, more work will be needed to fully understand the design space. The summary will discuss one potential advantage of low voltage electrostatic control.

	Zernike	Meaning	Correctable	Residual
	Polynomial		Portion	Portion
	1	Piston	1	0
	$\widehat{r}\sin heta$	y-axis tilt	$\widehat{r}\sin heta$	0
	$\widehat{r}\cos heta$	x-axis tilt	$\widehat{r}\cos heta$	0
	$\widehat{r}^2 \sin 2 heta$	Astigmatism (±45 degree axes)	$\widehat{r}^2 \sin 2\theta$	0
	$2\hat{r}^2 - 1$	Focus shift	$2\widehat{r}^2 - 1$	0
$4\mathrm{pt}$	$\widehat{r}^2\cos 2\theta$	Astigmatism (0 or 90 degree axes)	$\widehat{r}^2\cos 2 heta$	0
	$\hat{r}^3 \sin 3\theta$	Triangular astigmatism $(x$ -axis base)	$\hat{r}^3 \sin 3\theta$	0
	$(3\hat{r}^3 - 2\hat{r})\sin\theta$	Third-order coma (along x -axis)	$-2\widehat{r}\sin\theta$	$3\hat{r}^3 \sin \theta$
	$(3\hat{r}^3 - 2\hat{r})\cos\theta$	Third-order coma (along y -axis)	$-2\widehat{r}\cos\theta$	$3\hat{r}^3\cos\theta$
	$\hat{r}^3 \cos 3\theta$	Triangular astigmatism $(y$ -axis base)	$\widehat{r}^3 \cos 3\theta$	0
	$\widehat{r}^4 \sin 4\theta$		$\widehat{r}^4 \sin 4\theta$	0
	$(4\widehat{r}^4 - 3\widehat{r}^2)\sin 2\theta$		$-3\hat{r}^2\sin 2\theta$	$4\hat{r}^4\sin 2\theta$
	$6\hat{r}^4 - 6\hat{r}^2 + 1$	Third-order spherical	$-6 \hat{r}^2 + 1$	$6\hat{r}^4$
	$(4\hat{r}^4 - 3\hat{r}^2)\cos 2\theta$		$-3\hat{r}^2\cos 2\theta$	$4\hat{r}^4\cos 2\theta$
	$\widehat{r}^4 \cos 4\theta$		$\hat{r}^4 \cos 4\theta$	0
Ī	$\widehat{r}^5 \sin 5 heta$		$\hat{r}^5 \sin 5\theta$	0
	$(5\widehat{r}^5 - 4\widehat{r}^3)\sin 3\theta$		$-4\hat{r}^3\sin 3\theta$	$5\hat{r}^5\sin 3\theta$
	$(10\hat{r}^5 - 12\hat{r}^3 + 3\hat{r})\sin\theta$		$3\widehat{r}\sin\theta$	$(10\hat{r}^5 - 12\hat{r}^3)\sin\theta$
	$(10\widehat{r}^5 - 12\widehat{r}^3 + 3\widehat{r})\cos\theta$		$3\widehat{r}\cos\theta$	$(10\hat{r}^5 - 12\hat{r}^3)\cos\theta$
	$(5\hat{r}^5 - 4\hat{r}^3)\cos 3\theta$		$-4\hat{r}^3\cos 3\theta$	$5\hat{r}^5\cos 3\theta$
	$\widehat{r}^5\cos 5\theta$		$\hat{r}^5 \cos 5\theta$	0

Table 1. Wavefront correction by uniform pressure difference and boundary displacement *alone* (for wavefront aberration due to first 21 Zernike polynomials)

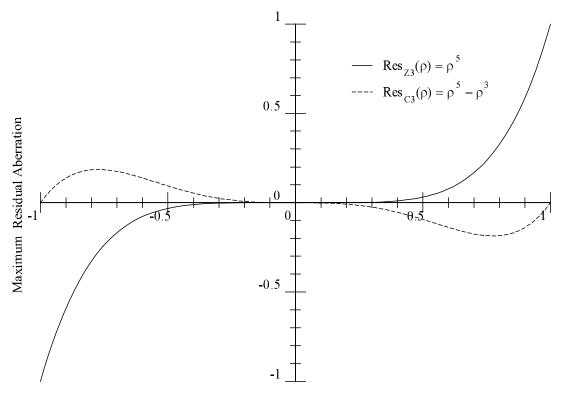
4. CHANGING THE RADIAL POLYNOMIAL BASIS

The Zernike radial polynomials may not be the most convenient basis representation of the wavefront aberration for membrane mirror analysis. For a given Zernike radial polynomial $\mathcal{Z}_m^n(\hat{r})$, and m > 2, we introduce a new polynomial $\mathcal{C}_m^n(\hat{r})$ defined by

$$C_m^n(\hat{r}) \equiv Z_m^n(\hat{r}) - \hat{r}^n, \qquad m > 2.$$
(8)

Using Malacara's definition¹⁰ of the Zernike radial polynomials, it is not difficult to show that when m = n, $C_m^m(\hat{r}) = 0$. More importantly, for $m \neq n$, each of these polynomials is zero at both the center $\hat{r} = 0$ and the edge $\hat{r} = 1$. The latter property leads us to refer to these polynomials as "clamped".

Consider, for example, the Zernike radial polynomial $\mathcal{Z}_5^3(\hat{r}) = 5\hat{r}^5 - 4\hat{r}^3$ appearing in the row second from the bottom of Table 1. The corresponding clamped polynomial is $\mathcal{C}_5^3(\hat{r}) \equiv \mathcal{Z}_5^3(\hat{r}) - \hat{r}^3 = 5\hat{r}^5 - 5\hat{r}^3 = 5\left(\hat{r}^5 - \hat{r}^3\right)$. Now, the residual aberration using the Zernike radial polynomial is proportional to \hat{r}^5 , as indicated in Table 1, but if the clamped polynomial is used, the residual aberration can be shown to be proportional to $\hat{r}^5 - \hat{r}^3$ (again, using only boundary displacement compensation). Figure 2 shows graphs of the residual aberrations for this example, and it is clear that in terms of the clamped polynomial the maximum residual to be corrected by electrostatic actuation is roughly 20% of that required using the Zernike polynomial. Thus, it would appear that by expressing the wavefront aberration in terms of clamped polynomials rather than Zernike radial polynomials, the amount of residual aberration to be compensated by electrostatic actuation could be reduced considerably, thereby reducing the actuation voltages required. The mathematical details of this analysis will be provided in the FY05 Laboratory Annual Report to AFOSR mentioned earlier.



Normalized Radial Coordinate ($\rho = r/a$)

Figure 2: Zernike residual (Z_5^3) verus clamped boundary residual

5. THE AIR GAP

The "air" gap has the potential to stiffen the system and significantly increase the fundamental frequencies, reducing ambient vibrations below optical tolerances. Again, this fortunate circumstance is made possible by the small dimensionless aspect ratio ℓ/a of air gap depth ℓ to membrane radius a. The fluid dynamics of the gap should account for the viscous nature of the gas both within the gap and on the surface of the membrane. The analysis is thus complicated as one must consider the coupling of the membrane dynamical equation (1) to the Navier-Stokes and mass conservation equations of viscous fluid dynamics. More needs to be done to gain a full understanding of the implications, but the initial results are promising. A complete summary of the analysis will be provided in the FY2005 AFOSR Laboratory Annual Report mentioned twice previously.

An indication of what might be expected of the resonant frequency when one particular limit is examined will help cast some light on the effect of the gap. We find that in the infinite-viscosity limit the radial and circumferential velocity components of the gas must vanish, assuming no-slip boundary conditions at the surfaces of the enclosure. The governing equations are then easily solved for the resonant frequencies, which are found to be given by the simple expression

$$f_{mn} = \frac{1}{2\pi} \omega_{mn} = \frac{1}{2\pi} \sqrt{\alpha_{mn}^2 \left(\frac{T}{\gamma_o a^2}\right) + \frac{p_0}{\gamma_o \ell}},$$

where p_0 is the scale factor used to scale the pressure difference (taken to be standard atmospheric pressure), and α_{mn} is the *m*th zero of the ordinary Bessel function J_n of order n. The additional term $(\frac{p_0}{\gamma_o \ell})$ under the radical indicates a stiffening effect of the gas, and is equivalent to the gas acting purely as a spring. A significant

amount of work is still required to sort out the impact of this and other results, and to put the entire design parameter space into perspective.

6. CONCLUSION

The PBS concept has the potential to overcome a large number of AO challenges that have been addressed in earlier sections, but one important feature deserves a forward looking statement. Since the electrostatic control voltage is on the order of 10 volts, the processor and a high density electrode assembly can be incorporated directly behind the membrane. This dramatically reduces the number of control wires to the AO and opens the possibility of using a set of densely packed actuators that will help smooth the actuation field, further reducing the influence effects of adjacent actuators.

One drawback of the PBS concept is the low dynamic operating bandwidth since the gap has substantially stiffened the system and the lower voltage actuators will supply only a limited amount of force. One possible way to overcome this restriction would be the introduction of a MEMS manifold to manage the airflow in the gap. One could envision a multi-channel micro-volume management mechanism incorporated into the electrostatic actuator plane to help improve the dynamic response.

Also, there may be a dramatic change in the membrane mechanics when a dielectric coating is added to the polyimide sheet. Since the thickness of the polyimide layer may be on the order of the thickness of the dielectric stack, and the elastic modulus of the polyimide is much less then the modulus of the dielectric materials, the resulting stiffness of the composite structure can be significantly higher. The composite layer may in fact be sufficiently stiff that the assumption that the system acts as a true membrane might be compromised. The use of control authorities 1 through 3 would in that case become even more important in the effort to lower the electrostatic control voltages and reduce the actuator influence effects.

ACKNOWLEDGMENTS

We thank three groups primarily responsible for financial support of the membrane mirror development, and subsequently the effort to develop the PBS: the Air Force Office of Scientific Research, the Directed Energy Directorate, and the Air Force Small Business Innovative Research program. Technical interaction with Professor Christopher Jenkins of the South Dakota School of Mines and Technology has been extremely helpful throughout the years, and is greatly appreciated. Finally, we are grateful to AFOSR for sponsoring a research effort at Virginia Tech that will continue the work of J. M. Wilkes who recently retired from government service.

REFERENCES

- R. Angel, J. Burge, K. Hege, M. Kenworthy, and N. Woolf, "Stretched membrane with electrostatic curvature (SMEC): A new technology for ultra-lightweight space telescopes," in *Proceedings of the SPIE, UV, Optical, and IR Space Telescopes and Instruments*, J. P. Breckenridge and P. Jakobsen, eds., *Proc. of SPIE* 4013, pp. 699-705, 2000.
- B. Stamper, R. Angel, J. Burge, and N. Woolf, "Flat membrane mirrors for space telescopes," in *Proceedings of the SPIE, Imaging Technology and Telescopes*, J. W. Bilbro, J. B. Breckinridge, R. A. Carreras, S. R. Czyzak, M. J. Eckart, R. D. Fiete, and P. S. Idell, eds., *Proc. of SPIE* 4091, pp. 126–136, 2000.
- 3. B. Stamper, R. Angel, J. Burge, T. Connors, B. Duffy, and N. Woolf, "Stretched membrane with electrostatic curvature (SMEC) mirrors for extremely large space telescopes," in *Proceedings of the SPIE*, Optical Manufacturing and Testing IV, H. P. Stahl, ed., Proc. of SPIE 4451, pp. 105–113, 2001.
- 4. S. Errico, R. Angel, B. Stamper, J. Burge, and T. Connors, "Stretched membrane with electrostatic curvature (SMEC) mirrors: A new technology for large lightweight space telescopes," in *Proceedings of the SPIE*, Highly Innovative Space Telescope Concepts, H. A. MacEwen, ed., Proc. of SPIE 4849, pp. 699-705, 2002.
- 5. R. P. Grosso and M. Yellin, "The membrane mirror as an adaptive optical element," *Journal of the Optical Society of America* **67**(3), pp. 399–406, 1977.

- 6. P. M. Morse, Vibration and Sound, The Acoustical Society of America, New York, second (paperback) ed., 1976.
- 7. D. K. Marker, J. M. Wilkes, R. A. Carreras, J. R. Rotgé, C. H. Jenkins, and J. T. Ash in Gossamer Spacecraft: Membrane and Inflatable Structures Technology for Space Applications, C. H. Jenkins, ed., AIAA Progress in Astronautics and Aeronautics 191, ch. 4, Fundamentals of Membrane Optics, pp. 111–201, American Institute of Aeronautics and Astronautics, 2001.
- 8. D. K. Marker, R. A. Carreras, J. R. Rotgé, and C. H. Jenkins, "An update on membrane telescopes used in ground based astronomy," 2001. Proceedings of the AMOS, Wailea, Maui, Hawaii.
- 9. K. F. Graff, Wave Motion in Elastic Solids, Ohio State University Press, Columbus, OH, 1975.
- 10. D. Malacara, ed., Optical Shop Testing, John Wiley & Sons, New York, second ed., 1992.